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SOME RECENT MATHEMATICAL WORKS.

Correspondance d'Hermite et de Stieltjes. Edited by B. Baillaud and H. Bourget. Vol. ii. Pp. viii+404. (Paris: Gauthier-Villars, 1905.) Price 16 francs.

G. Lejeune Dirichlet's Vorlesungen über die Lehre von den einfachen und mehrfachen bestimmten Integralen. By G. Arendt. Pp. xxiv+476. (Brunswick: Friedrich Vieweg and Son, 1904.) Price 12 marks.

Le Calcul des Résidus et ses Applications à la Théorie des Fonctions. By Ernst Lindelöf. Pp. viii+144. (Paris: Gauthier-Villars, 1905.) Price 3.50 francs.

Les Principes des Mathématiques. By Louis Couturat. Pp. viii+342. (Paris: Félix Alcan, 1905.)

Méthodes de Calcul graphique. By Frederico Oom. Pp. 26; with 4 plates. (Lisbon: Imprimerie nationale, 1905.)

Volume and Surface Integrals used in Physics. By J. G. Leathem. Pp. 48. (Cambridge: University Press, 1905.) Price 2*s.* 6*d.* net.

Recherche sur les Champs de Force hydrodynamiques. By V. Bjerknes. Reprinted from *Acta Mathematica*, vol. xxx. Pp. 146.

Sur la Recherche des Solutions particulières des Systémes différentiels et sur les Mouvements stationnaires. By T. Levi Civita. Pp. 40. (Warsaw: J. Sikorskiego, 1906.)

ONE of the most noticeable features of recent times has been the increasing interest taken in the history of mathematics. That two international congresses—the historical and the mathematical—have devoted separate sections to this study is, let us hope, a stepping-stone towards the realisation of the resolutions passed at both congresses in favour of the establishment of chairs of mathematical history in the leading universities of the Continent and America, and possibly even Great Britain.

Reference has previously been made to the first volume of the interesting correspondence between Hermite and Stieltjes. The second volume, covering the period 1889–1894, is no less delightful reading than the first. Every letter fills up some gap in the reader's mathematical knowledge, either by introducing him to some little-known proposition or by presenting some well-known result in a new aspect. An appendix contains four letters addressed by Stieltjes to Prof. Mittag-Leffler in 1885–1887 dealing with Riemann's Zeta function. These letters afford a striking insight into the difficulties experienced by Stieltjes in his efforts to master Riemann's works and his ingenuity in devising alternative methods. The present volume contains a portrait of Hermite and the facsimile of a manuscript by Stieltjes.

The historic spirit has further shown itself in Prof. Arendt's issue of the nearest possible approach to a verbatim report of the lectures on definite integrals as given by Dirichlet at Berlin in 1854. It is true, as the author points out, that the lectures which Dirichlet gave on the same subject at Göttingen four years later formed the basis of the well-known treatise by Gustav Ferdinand Meyer, but it appears that the notes on which Meyer's account was based were far from complete, and it was necessary to spend considerable time in filling up the gaps in the reasoning, and, moreover, the object was to give a complete treatment of the subject rather than an exact account of the lectures. Prof. Arendt, on the other hand, has compiled the present work from a set of notes mostly transcribed on the actual dates of the lectures. The course covers a branch of mathematics well known to the average student, namely, the definition of an integral and its connection with summation, the theorems on change of limits and differentiation of integrals, the evaluation of the ordinary well-known definite integrals, the Beta and Gamma functions, transformation of multiple integrals, the attractions of ellipsoids, and applications to harmonic and hypergeometric series. The notes at the end afford evidence of the care with which the original manuscript has been followed; where any divergence has been necessary the changes are carefully pointed out; the only important innovation, however, is the introduction of the modern notation $[a]$, which greatly simplifies certain formulæ. At the present time these lectures of Dirichlet make an excellent text-book, and an interesting historical comparison may be made between the present course and Kronecker's lectures delivered at the same university about thirty years later.

Another prominent place among the "classics" must be assigned to Prof. Ernst Lindelöf's charming exposition of Cauchy's calculus of "residues." This is the eighth of a series of monographs on the theory of functions appearing under the editorship of Prof. Émile Borel. In Prof. Lindelöf, Cauchy's ideas have found an able exponent, and from a detailed study of a number of papers, including some of Cauchy's little-known writings lent for the purpose by Prof. Mittag-Leffler, the author has produced a treatise in which the simplicity and perfection of this important method of analysis are well shown. Of the applications, those in the second chapter are mainly due to Cauchy. The third shows how certain formulæ of summation can be immediately deduced from the same principle, while in the fourth it is shown how this method of treatment greatly simplifies the study of the Gamma function and of Riemann's function. Of the importance of the latter application the difficulties of Stieltjes already referred to give sufficient proof, and on the other hand the name of Stieltjes figures conspicuously in the discussion of Stirling's series, in connection with which Prof. Lindelöf contributes several new results and proofs. Finally we have a general account of certain modern results relating to functions defined by Taylor's series, thus bringing into one small volume a general survey of the recent as well as the original developments of Cauchy's method. The book includes new matter for which the author is himself responsible as regards methods of treatment no less than as regards results.

A second line of modern mathematical development consists in the attempt to probe ever deeper and deeper into the foundations of mathematics. In France,

where everything mathematical is as popular as it is unpopular in England, the philosophy of mathematics has taken such hold of public thought that the multiplication of books on the subject has in a small way resembled the multiplication of school geometries with us. But a philosophical treatise stands on a very different level from a mere examination text-book, and if we have been somewhat severe in the past in our criticisms of the work of isolated writers in France, it was felt that what was wanted was something more than a number of isolated writings, each, from the nature of the case, presenting the views of one individual without much reference to the subject and its literature considered as a whole. The opening words of M. Couturat's preface, "The present book has no pretension to originality, and this is precisely what ought to recommend it to the reader," show that the author has been at great pains to fill the want. His book is to a large extent based on Mr. Bertrand Russell's English treatise with the same title, and is intended to provide a *résumé* of our present knowledge on the philosophy of mathematics. It is somewhat remarkable that up to the middle of last century logic and mathematics were regarded as essentially distinct, and it was largely the result of the labours of Boole, Peano, Cantor, and others that led to the gap being filled and to the opening out of what has proved to be one of the most fertile regions of modern thought. The complete rapprochement owes its existence very largely to the symbolical logic or "logistic" of Peano, and leads to the conclusion that mathematics is entirely and exclusively founded on the principles of logic. This view is, as M. Couturat shows, diametrically opposed to the philosophy of Kant of which a summary has been given in the appendix. It need hardly be said that Russell's treatment is in many places closely followed, and it is the author's hope that the book will induce French writers to contribute to our knowledge of mathematical philosophy in a way that has not been done hitherto.

A perusal of Mr. Oom's pamphlet will convince any reader that however much has been done elsewhere in facilitating calculations by the introduction of graphical methods, the observatory of Lisbon under the directorship of Vice-Admiral Campos Rodrigues has made a number of very distinct advances. For the correction of level and deviation error diagrams are used, as also for the corrections due to precession, and a still happier thought is the construction of slide rules for the performance of addition operations other than the addition of logarithms performed by the ordinary slide rule. Thus, for example, a slide rule graduated in reciprocals is used to work out relations between the conjugate foci of a lens; another, graduated in squares, is applicable to quantities connected by the relation between the sides of a right-angled triangle, and is particularly useful for calculations of probable error, and so on. Possibly some reader of NATURE will write and say that these slide rules have been in existence previously. In any case they are worthy of note, and M. Rodrigues appears to have devised them "off his own bat."

The issue of a series of "Cambridge Tracts in Mathematics and Mathematical Physics" under the able editorship of Messrs. Leathem and Whittaker affords evidence of the activity of the younger generation of Cambridge mathematicians. The idea is a good one. Many people have ideas about the best methods of treating some particular piece of work, which do not cover a sufficiently wide range to form a book, and are unsuitable for publishing and possibly burying in a volume of transactions. So much is this the case that we should not be surprised if Mr. Leathem finds himself besieged by tracts submitted for publication. His own contribution deals with an important point. Many physicists in the solution of problems have had to transform volume and surface integrals in a way that they either have known or ought to have known was not perfectly rigorous, but with the knowledge that the results would be all right; this particularly applies to integrals extending to infinity, and we have here in a convenient form a study of these transformations in their mathematical aspect. In discussing the application of infinitesimal analysis to potential properties of bodies of discontinuous structure, Mr. Leathem introduces the notion of *physical smallness*. The term is, perhaps, not altogether a happy one, as physics concerns itself not only with bodies of finite size, but with molecules which are of a higher order of smallness than the elements contemplated. The important point is that the applications of differential equations are based on the consideration of elements which for purposes of analysis may be regarded as infinitesimal, but which may be regarded, on the other hand, as infinitely great compared with the dimensions of molecular structure. It would be better to call such elements "differential elements" since they represent the $dx\ dy\ dz$ of the formulæ. The careful discussion of the legitimacy of the assumptions involved, as given by Mr. Leathem, is important, as we often find unscientific writers announcing as a great discovery the view that there is no such thing as temperature, quite forgetting that the notion of "temperature at a point" stands on much the same footing as that of "density at a point" or, indeed, many other similar concepts without which the study of mathematical physics would not have made the progress that it has made.

The series of papers and books by C. A. Bjerknes the father and V. Bjerknes the son well illustrate the proper spirit of scientific inquiry as opposed to the spirit of the unscientific faddist whose rejected addresses give so much trouble to reviewers. The discussion of the fields produced by bodies moving in fluids, if it has not given us a new theory of matter has certainly greatly helped us to understand the lines on which such theories should be laid down. The elder Bjerknes confined his attention to solid spheres moving in liquid; in the present instance, "bodies" are represented by portions of fluid differing from the remainder by the fact that in the latter the equations take a simple form. But is not this merely the vortex atom theory? It is true that on pp. 134–7 Prof. Bjerknes compares his results with those of von Helmholtz and Lord Kelvin, and points out the differences in his method of treatment, but all these investigations are

only different developments of the same fundamental ideas.

In recent years, Prof. Levi Civita has published a number of papers in the *Atti dei Lincei* dealing with particular solutions of the equations of dynamics, and in especial with stationary motions. At the invitation of Prof. Dickstein he has now prepared a simplified account of these researches for the transactions of the "Prac matematyczno fizyczynych," published at Warsaw. The original starting point of the investigation was the method of ignoration of coordinates, but the conclusions have now been shown to be results of a general principle applicable to any system of ordinary differential equations. They form a development of the work of Routh, and the stationary motions investigated by the author of "Rigid Dynamics" are shown to belong to a particular class to which Prof. Levi Civita gives the name of "mouvements à la Routh." G. H. B.

A TREATISE ON CHEMISTRY.

A Treatise on Chemistry. By Sir H. E. Roscoe, F.R.S., and C. Schorlemmer, F.R.S. Vol. i. The Non-Metallic Elements. New edition, completely revised by Sir H. E. Roscoe. Pp. xii+931. (London: Macmillan and Co., Ltd., 1905.) Price 21s. net.

SIR HENRY ROSCOE is to be heartily congratulated by all chemists on the appearance of a new edition of the first volume of Roscoe and Schorlemmer's "Treatise on Chemistry." This volume deals chiefly with the non-metallic elements, and is now in its third edition.

Many chemists remember the interest which the first appearance of this volume excited in 1877. Printed in large clear type, with excellent illustrations, it was recognised both here and on the Continent as a clear and readable account of the facts relating to the chemistry of the non-metallic elements. If the student failed to find in it any new light on the obscurities of chemical theory, he at any rate was put in possession, not merely of the facts, but of the facts stated with a due regard for the history of their discovery which was then and is still foreign to the ordinary "handbook." There were, moreover, many experimental details of service to workers in the laboratory recorded in the volume which were at that time not easily accessible to the ordinary student. During the nearly thirty years which have elapsed since the first edition appeared, many treatises have been published in other languages, notably in German, but the treatise of Roscoe and Schorlemmer still retains a certain individuality for which it will be valued.

In preparing this edition Sir Henry Roscoe has had the valuable assistance of several collaborators with special knowledge, and their handiwork is to some extent evident in the different literary treatment which may be discerned in various sections of the book.

The first section of the volume relates to the general principles of the science, including a description of the properties of gases and liquids, and a very intelligible account of the development of the atomic theory. A clear sketch is given of the theory of electrolytic dissociation. This portion of the volume would have been improved by some concrete illustrations of the methods of determining atomic weights. As it is, the reader must be very much at sea in understanding what this constant actually means, apart from the implications of the atomic theory.

The remainder of the book is occupied with an account of the properties and modes of preparation of each of the non-metallic elements and their chief compounds. The history of each element is succinctly and well described, and important industrial applications are also alluded to. There is an excellent account of the modern manufacture of illuminating gas and of acetylene, as well as of the commercial processes adopted for the production of a number of the elements and their compounds which find industrial uses.

There is also a very complete account of the preparation and properties of the new gases of the atmosphere, argon, &c., which, while interesting, does not throw any new light on the obscure chemical relationships of these elements. In this connection the absence of an account of the periodic classification under the general principles of the science is specially felt. It would have been better to have included in the first part of this volume a complete consideration of the general principles of chemistry, including the determination of atomic weights, instead of reserving the discussion of the periodic classification and other matters of principle for the subsequent volume relating to the metals.

When the first edition of this volume appeared, many of the lecture experiments described were new, and were of interest and value to the teacher. A number of these are now generally familiar, whilst some of those still described have since been improved upon. This feature is indeed no longer a striking one in the book. Very few new lecture or laboratory experiments are included. The teaching of chemistry is, however, no longer conducted on the old lines, and perhaps the teacher would not now look to a treatise of this kind for this information. The fact that in some sections of the work pains are taken to describe fully striking lecture experiments whilst in other and newer sections this aspect is entirely neglected is a defect in the general plan of the book which might be remedied in future editions.

This raises the question as to the characters which such a work as this should possess to be of real utility at the present day. Handbooks and text-books of chemistry for the teacher abound, many of them excellent as practical guides to the work of the lecture room and laboratory. Then there are more ambitious works purporting to be of the nature of treatises. These, however, are too often ill-assorted and ill-considered collections of the facts and theories of chemistry utterly lacking in those literary qualities without which no work of the kind can expect to appeal to the general reader or to take any permanent place in the literature of the science. There is still room for a treatise in the broad sense of the word, in which the facts and doctrines of modern chemistry are expounded in a lucid manner free from the details and technicalities which are essential in a handbook or